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(June 24, 2000)

We calculate the reheating temperature in the central region of very high energy hadronic collisions assuming saturation of the number of secondary partons. From the condition that (approximate) kinetic equilibrium establishes when the rescattering rate equals the Bjorken/Hubble expansion rate we find that the thermalization time increases with energy because of asymptotic freedom. Furthermore, the reheating temperature does not increase far beyond the QCD scale Λ_{QCD}, no matter how high the collision energy. That could mean that hadronic collisions at any energy can only reach temperatures up to a few times Λ_{QCD}, not more. For collisions of very heavy nuclei, the maximum reachable temperature occurs in the energy range of BNL-RHIC to CERN-LHC.

According to the standard Big Bang model [1] the temperature T in the early universe at times $t < 10^{-6}$ seconds must have been on the order of Λ_{QCD} and larger. Theoretical arguments [2] and lattice gauge theory results [3] suggest that at such high temperatures the thermodynamically stable state of QCD is different from that at $T = 0$, i.e. the theory of strong interactions exhibits a phase transition to a new state called "Quark Gluon Plasma" (QGP).

It is currently sought to recreate that hot primordial state of strongly interacting matter in the laboratory by colliding protons or heavy ions at very high energies [4]. Cosmic ray collisions with our atmosphere might be another promising testing ground for QGP formation [5]. In such high-energy hadronic collisions the density of partons released in the central region becomes very large, see below. It is hoped that subsequent interactions evolve the dense parton state towards local kinetic equilibrium and form the QGP state. In this Letter we calculate the reheating temperature in very high energy hadronic collisions and discuss its evolution with increasing center of mass energy.

Within the collinear factorization approach [6] the number of minijets at central rapidity, $y \simeq 0$, and with transverse momentum p_T above the scale p_0 is given by

$$\frac{\mathrm{d}N}{\mathrm{d}y}(p_0) = K T_{AA}(b=0) \int_{p_t>p_0} \mathrm{d}^2 p_T \int \mathrm{d}x_a \mathrm{d}x_b \\ G\left(x_a, p_T^2\right) G\left(x_b, p_T^2\right) \frac{\hat{s}}{\pi} \frac{\mathrm{d}\sigma}{\mathrm{d}\hat{t}} \delta\left(\hat{s}+\hat{t}+\hat{u}\right) \quad . \tag{0.1}$$

$\hat{s}$, $\hat{t}$, $\hat{u}$ are the usual Mandelstam variables of the parton-parton scattering process, and $\mathrm{d}\sigma/\mathrm{d}\hat{t}$ denotes the hard-scattering differential cross section in lowest order of perturbative QCD. $G(x, p_t^2)$ denotes the LO parton distribution function in the proton or nucleus, respectively. At high energies or small x it is sufficient for a qualitative understanding to focus on gluon-gluon processes. The phenomenological factor $K = 2$ is meant to account for NLO corrections to the above expression. The overlap function $T_{AA}(0) = A^2/\pi R_A^2$ determines the number of binary parton collisions within the Glauber approach, where $R_A \simeq 1.1 A^{1/3}$ fm denotes the effective hard-core radius for central collisions of mass A nuclei.

For very large p_0 the system of produced gluons is rather dilute. As p_0 decreases, however, the density of gluons liberated from the proton/nuclear wave function increases rapidly, which is caused by the increase of $G(x, p_t^2)$ at small x. One may conjecture that at some transverse momentum scale $p_0 = p_{\rm sat}$ the phase-space density of produced gluons essentially saturates due to the fact that the $\mathrm{d}N/\mathrm{d}y(p_{\rm sat})$ partons each having a transverse "area" $\pi/p_{\rm sat}^2$ fill the available transverse area πR_A^2 of the incoming beam [7]. In other words, the saturation scale $p_{\rm sat}$ is determined by the condition $\mathrm{d}N/\mathrm{d}y(p_{\rm sat}) = p_{\rm sat}^2 R_A^2$. This idea has recently been employed to calculate the saturation scale $p_{\rm sat}$ as function of mass number A and center of mass energy $\sqrt{s}$ [8]. Moreover, if $p_{\rm sat}$ is indeed the only scale in the problem (at asymptotically high energies), the space-time picture of initial parton production is completed by noting that those partons have to be released on the proper time hyperbola [9] $\tau_0 = 1/p_{\rm sat}$, by virtue of the uncertainty principle. In Bjorken's space-time picture the longitudinal streaming of the produced partons is described by a FRW metric in the space-time variables τ (proper time) and η (space-time rapidity) with (longitudinal) scale factor $a = \tau$. Thus, the line element in the (τ, η)-plane is $\mathrm{d}s^2 = \mathrm{d}\tau^2 - \tau^2 \mathrm{d}\eta^2$, corresponding to a longitudinal expansion rate (Hubble constant) $\Gamma_{exp} = \dot{a}/a = 1/\tau$. Thus, it is clear that as the scale $p_{\rm sat}$ grows, the density of produced partons increases $\sim p_{\rm sat}^3$, while on the other hand their cross-section decreases as $\sim 1/p_{\rm sat}^2$, and at the same time the expansion rate also increases as $\Gamma_{exp}(\tau_0) = p_{\rm sat}$.

More precisely, the scattering rate is determined by the transport cross-section

$$\sigma_t = \int \mathrm{d}\Omega \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \sin^2\theta_{c.m.} \quad . \tag{0.2}$$

To leading logarithmic order, taking into account elastic $gg \to gg$ scattering including screening of long-wavelength static color fields by the other gluons present at the scale $p_{\rm sat}$ leads to

$$\sigma_t = \frac{9}{2}\frac{4\pi\alpha_s^2(\overline{s})}{\overline{s}}\log\frac{1}{\alpha_s(\overline{s})} \quad . \tag{0.3}$$

The average energy per produced gluon is $\overline{s} = 2C_1^2 p_{\rm sat}^2$, with $C_1 = \mathcal{O}(1)$ being roughly constant, independent of the scale $p_{\rm sat}$ [8]. Therefore, noting that the comoving gluon density at time $\tau_0 = 1/p_{\rm sat}$ is $\rho_0 = p_{\rm sat}^3/\pi$, we obtain for the ratio of scattering rate to expansion rate at that time

$$\frac{\Gamma_{scatt}}{\Gamma_{exp}} = \tau_0\sigma_t\rho_0 = \frac{9\alpha_s^2(p_{\rm sat}^2)}{C_1^2}\log\frac{1}{\alpha_s(p_{\rm sat}^2)} \quad . \tag{0.4}$$

Thus, the Bjorken/Hubble expansion rate $1/\tau_0 = p_{\rm sat}$ removes the last remaining power of $p_{\rm sat}$ when taking the ratio $\Gamma_{scatt}/\Gamma_{exp}$. However, there does remain a dependence on the scale $p_{\rm sat}$ which is due to the running of the QCD coupling. Thus, because of asymptotic freedom [10], the ratio (0.4) logarithmically approaches zero as $p_{\rm sat} \to \infty$, meaning that the initial kinetic equilibration rate goes down as the saturation scale increases. Note that this observation relies essentially on the fact that within the saturation model the dominant scale $p_{\rm sat}$ entering eq. (0.1) *evolves* with A and $\sqrt{s}$. The conclusion is different if one employs a constant energy independent cutoff p_0.

The next step is to study the subsequent evolution of the gluon minijets. Based on the above arguments we shall assume for the moment that the partons are actually close to free-streaming at time τ_0. (However, we also perform numerical calculations without that assumption, see below.) In that regime the average energy per parton, and thus the transport cross-section (0.3), are almost constant, and thus the scattering rate $\Gamma_{scatt}(\tau) = \sigma_t\rho(\tau) \propto 1/\tau$, since the comoving gluon density for the above-mentioned metric decreases inversely proportional to the scale factor a. One can thus ask when the time-integrated scattering rate equals unity, i.e. the (average) time when one collision occured [11,12]. This happens at

$$\phi(\tau) = 1 \to \frac{\tau_{N=1}}{\tau_0} = \exp\left(\frac{C_1^2}{9\alpha_s^2\log(1/\alpha_s)}\right) \quad , \tag{0.5}$$

where

$$\phi(\tau) \equiv \int_{\tau_0}^{\tau} {\rm d}\tau'\Gamma_{scatt}(\tau') \simeq \frac{9\alpha_s^2\log(1/\alpha_s)}{C_1^2}\log\frac{\tau}{\tau_0} \tag{0.6}$$

denotes the integrated collision rate. The last expression holds close to the free streaming regime ("Knudsen limit"), where the number of scatterings increases only logarithmically with time. With the parametrization $C_1(p_{\rm sat})$ given in [8] one actually obtaines $\tau_{N=1} \simeq 1$ fm in the BNL-RHIC to CERN-LHC energy region, increasing slowly with energy. However, it turns out that the ratio of the scattering and expansion rates[1] is smaller than unity (asymptotically approaching zero). Thus, it appears that the few rescatterings among the almost freely streaming produced partons are not efficient in making the momentum distribution in the locally comoving frame more isotropic (or to keep it isotropic, in case of an isotropic initial condition; see below). Rather, the red-shift due to the expansion dominates and "squeezes" the momentum-space distribution in longitudinal direction. Therefore, rather than taking $\tau_{N=1}$ from eq. (0.5) to be the thermalization time, we define it by the condition (see also [1])

$$\frac{\Gamma_{scatt}(\tau_{th})}{\Gamma_{exp}(\tau_{th})} = 1 \ . \tag{0.7}$$

Of course, the latter condition can only be ever met if we account for the few residual collisions which occur even at asymptotically high energies, see eq. (0.4), for otherwise the ratio (0.7) is constant (time independent) and smaller than unity.

Thus, we make the *Ansatz* that the energy density (in the comoving frame) decreases slightly faster than in the free streaming regime,

$$\frac{\tau\epsilon}{\tau_0\epsilon_0} = \left(\frac{\tau_0}{\tau}\right)^{\delta} \quad , \tag{0.8}$$

with the parameter $\delta \ll 1$ parametrizing the slight deviation from free streaming. This ansatz for the energy density corresponds to a scattering rate

$$\Gamma_{scatt} = \frac{1}{\tau_{scatt}} = \frac{9\alpha_s^2 p_{\rm sat}}{C_1^2}\left(\frac{\tau_0}{\tau}\right)^{1-2\delta}\log\frac{1}{\alpha_s} \ , \tag{0.9}$$

where we introduced the scattering-time τ_{scatt}. It is actually possible to calculate δ, for example by employing the Boltzmann equation in relaxation time approximation [13],

$$p\cdot\partial f(p,x) = \frac{p\cdot u}{\tau_{scatt}}\left(f_{eq}(p\cdot u) - f(p,x)\right) \ . \tag{0.10}$$

u^μ denotes the four-velocity of the comoving frame and f_{eq} is the equilibrium phase space distribution, a Bose distribution in case of gluons, towards which the evolution is eventually supposed to converge. Since τ_{scatt} is very large initially, the driving force to equilibrium is rather small. Taking the first moment of eq. (0.10), one arrives at [13]

$$e^{\phi}\frac{\tau\epsilon}{\tau_0\epsilon_0} = 1 + \int_0^{\phi}{\rm d}\phi' e^{\phi'}\frac{\tau'(\phi')\epsilon(\phi')}{\tau_0\epsilon_0}h\left(\frac{\tau'(\phi')}{\tau(\phi)}\right) \ . \tag{0.11}$$

In deriving this equation we assumed that the initially produced partons have vanishing longitudinal momentum spread in the comoving frame. We shall discuss the

[1]Eq. (0.4) with the scale of the running coupling being $\overline{s}$ instead of $p_{\rm sat}^2$, which is the same in leading logarithmic approximation.

opposite scenario of isotropic initial momentum distribution below. The function $h(x)$ appearing in (0.11) is given by

$$h(x) = \frac{1}{2}\left(x + \frac{\arcsin\sqrt{1-x^2}}{\sqrt{1-x^2}}\right) \ . \tag{0.12}$$

The integral-equation (0.11) could be studied numerically, cf. e.g. [13–15]. However, let us first proceed analytically in order to extract the general asymptotic behavior as function of energy or $p_{\rm sat}$. To that end we expand (0.11) to first order in ϕ (which is the integrated scattering rate). We find

$$\frac{\tau\epsilon}{\tau_0\epsilon_0} = 1 + \left(\frac{\pi^2}{16} - \frac{3}{4}\right)\phi + \mathcal{O}\left(\phi^2\right) \ . \tag{0.13}$$

We can use (0.9) to expand this result to first order in δ, and upon comparison to the original ansatz (0.8), also expanded to $\mathcal{O}(\delta)$, we see that

$$\delta = \left(\frac{3}{4} - \frac{\pi^2}{16}\right)\frac{9\alpha_s^2(p_{\rm sat})}{C_1^2}\log\frac{1}{\alpha_s(p_{\rm sat})} \ , \tag{0.14}$$

which indeed approaches zero as $p_{\rm sat} \to \infty$. Strictly speaking this solution is only valid as long as the integrated number of scatterings (0.5) is much smaller than one. However, one may also extrapolate the time-evolution (0.8) to times where the scattering and expansion rates, respectively, become comparable. This of course assumes that the rate of departure from free streaming (our parameter δ) does not "explode".

In that case the thermalization time as defined in eq. (0.7) is

$$\frac{\tau_{th}}{\tau_0} = \left(\frac{C_1^2}{9\alpha_s^2(p_{\rm sat})\log 1/\alpha_s(p_{\rm sat})}\right)^{\frac{1}{2\delta}} \ , \tag{0.15}$$

and from the energy density at that time, see eq. (0.8), we obtain the "reheating temperature"

$$T(\tau_{th}) = \left(\frac{30\epsilon(\tau_{th})}{16\pi^2}\right)^{1/4} \tag{0.16}$$

$$= \left(\frac{30C_1p_{\rm sat}^4}{16\pi^3}\right)^{1/4} \times \left(\frac{9\alpha_s^2(p_{\rm sat})\log 1/\alpha_s(p_{\rm sat})}{C_1^2}\right)^{\frac{1+\delta}{8\delta}} \ . \tag{0.17}$$

Since $\delta \to 0$ as $p_{\rm sat} \to \infty$, we find that asymptotically $T \to 0$. The entropy at τ_{th} is

$$\frac{\tau_{th}s(\tau_{th})}{\pi R_A^2} = \frac{4}{3}\frac{\epsilon(\tau_{th})}{T(\tau_{th})}\tau_{th} = \frac{4}{3}p_{\rm sat}^2\left(\frac{16C_1^3}{30\pi}\right)^{1/4} \times \left(\frac{9\alpha_s^2(p_{\rm sat})\log 1/\alpha_s(p_{\rm sat})}{C_1^2}\right)^{\frac{3\delta-1}{8\delta}} \ , \tag{0.18}$$

where $s \equiv 4\epsilon/3T$ denotes the entropy density in the comoving frame. One observes that as the scale $p_{\rm sat}$ increases more and more entropy is produced until thermalization is eventually reached.

To obtain some estimates for the reheating temperature and thermalization time we employ the parametrizations of $C_1(A,\sqrt{s})$ and $p_{\rm sat}(A,\sqrt{s})$ from ref. [8], i.e.

$$p_{\rm sat} = 0.208A^{0.128}\sqrt{s}^{0.191}$$
$$C_1 = 1.3A^{-0.007}\sqrt{s}^{0.021} \ , \tag{0.19}$$

where $p_{\rm sat}$ and $\sqrt{s}$ are in units of GeV. For the running coupling constant we use the expression at the one-loop level with $N_f = 3$, $\Lambda_{QCD} = 0.2$ GeV, and evaluate at the scale $p_{\rm sat}$. From there we can compute δ which indeed turns out to be small, $\delta \approx 0.09$ for Au+Au at RHIC energy, and even smaller at higher energies.

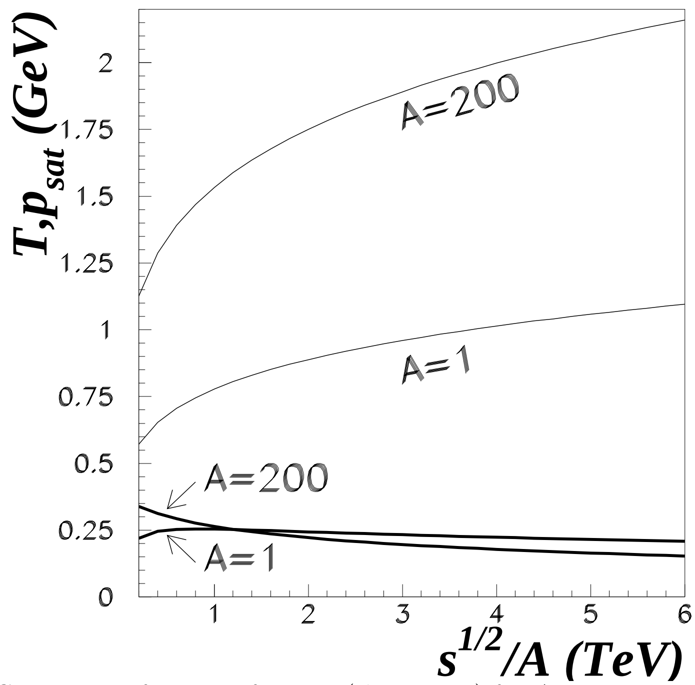

FIG. 1. $p_{\rm sat}$ as function of energy (thin curves) for $A = 1$ and $A = 200$, and the reheating temperature (thick curves).

Fig. 1 shows the increase of the saturation scale and the corresponding evolution of the temperature with energy (for $A = 1$ and $A = 200$). For $A = 1$ we see that $T \approx 220$ MeV at $\sqrt{s} = 200A$ GeV, then increasing slightly before bending over into an asymptotic decrease. For very heavy ions with mass number $A = 200$ we obtain $T \approx 340$ MeV at $\sqrt{s} = 200A$ GeV, and dropping right away towards higher energy. Also, despite the rather strong A-dependence of the saturation scale $p_{\rm sat}$, the reheating temperature is much less affected by the mass number of the projectile and target, in particular at high energy. As a side-remark we mention that for the parametrization (0.19) the thermalization time is $\tau_{th} \approx 1.5$ fm for $A = 200$ and $\sqrt{s} = 200A$ GeV, but according to eq. (0.15) increases to ≈ 170 fm at CERN-LHC energy, $\sqrt{s} = 5.4A$ TeV.

Although the quoted numbers for T and τ_{th} are actually close to previous numerical estimates [14–16] for heavy-ion collisions at RHIC energy, one should be aware that they are just rough estimates. For example, changing the parametrizations (0.19) one can make T increase or decrease by factors of $2-5$. Obviously, since T is proportional to $(\alpha^2/C_1^2)^{1/8\delta}$, small changes in δ have significant effect. Nevertheless, *asymptotically* δ decreases with increasing energy if the scale $p_{\rm sat} \to \infty$, and consequently $\tau_{th} \to \infty$ and $T \to 0$.

The analytic estimate presented above is only strictly valid if starting close to the free-streaming regime, and only up to the point where the accumulated number of collisions $\phi \sim 1$. However, the thermalization condition (0.7) is fulfilled at much later times, where $\phi > 1$. Thus, it is important to check that the conclusions are not artifacts of our naive extrapolation. To that end we have also solved eq. (0.11) numerically, with the following modifications.

First, we allow for the fastest possible thermalization by chosing the momentum distribution (in the comoving frame) of the initially produced partons to be isotropic. This amounts to replacing the 1 on the right-hand-side of eq. (0.11) by $h(\tau_0/\tau)$. Furthermore, we no longer employ the $\delta \to 0$ limit of ϕ, eq. (0.6), but rather integrate the scattering rate numerically. Γ_{scatt}, in turn, is calculated in each time-step via

$$\Gamma_{scatt}(\tau) = \frac{18\pi\alpha_s^2(p_{\rm sat})}{\overline{s}}\rho \log\frac{1}{\alpha_s(p_{\rm sat})} = \frac{p_{\rm sat}^5}{\tau^3\epsilon(\tau)^2}\frac{C_1^2}{\pi^2}\Gamma_{scatt}(\tau_0)\ . \qquad (0.20)$$

The average energy per parton at time τ is $\sqrt{\overline{s}/2} = \epsilon(\tau)/\rho(\tau)$. For our choice of metric the comoving parton density behaves as $\rho(\tau) = \rho_0\tau_0/\tau$, where $\rho_0 = p_{\rm sat}^3/\pi$ denotes the initial parton density. The integral-equation (0.11) now determines the self-consistent evolution of the parton energy density. A form suitable for numerical integration is

$$\frac{\tau\epsilon(\tau)}{\tau_0\epsilon_0} = h\left(\frac{\tau_0}{\tau}\right)\exp\left(-\int_{\tau_0}^{\tau} {\rm d}\tau'\Gamma_{scatt}(\tau')\right) + \int_{\tau_0}^{\tau}{\rm d}\tau'\Gamma_{scatt}(\tau')\frac{\tau'\epsilon(\tau')}{\tau_0\epsilon_0}h\left(\frac{\tau'}{\tau}\right)\times \exp\left(-\int_{\tau'}^{\tau}{\rm d}\tau''\Gamma_{scatt}(\tau'')\right) \qquad (0.21)$$

with Γ_{scatt} given in (0.20). This evolution is self-consistent (see also refs. [14,15]) in the sense that the evolution of the energy density in each time-step is governed by the scattering rate, which in turn is determined by the energy density. The evolution of the scale factor $a = \tau$ (or of the expansion rate) for purely longitudinal Bjorken expansion fortunately decouples, and is already accounted for in eq. (0.21): the local energy density is multiplied by the scale factor throughout the equation. The problem would be considerably more complicated if one allowed for transverse expansion as well, because in that case the expansion rate *does* depend on the local energy density and in turn feeds back to the evolution of $\epsilon(\tau)$ [17].

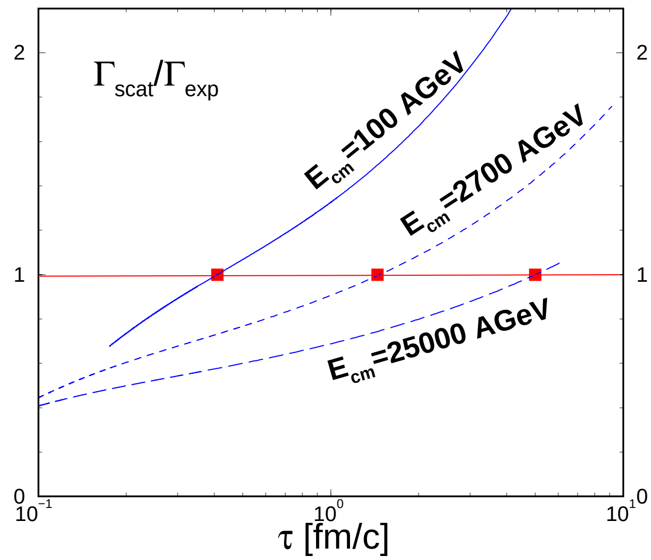

FIG. 2. The ratio of the scattering and expansion rates as function of time for three different center of mass energies $E_{cm} = \sqrt{s}/2$ and $A = 200$ nuclei.

Aside from $\epsilon(\tau)$ we have also determined the temperature and entropy corresponding to local thermal equilibrium via eq. (0.16) and

$$\frac{\tau s}{\tau_0 s_0} = \left(\frac{\epsilon}{\epsilon_0}\right)^{3/4}\frac{\tau}{\tau_0}\ . \qquad (0.22)$$

In Fig. 2 we show the scattering rate divided by the Bjorken/Hubble expansion rate at three different energies. As already mentioned above, at the initial time that ratio decreases for $p_{\rm sat} \to \infty$ due to asymptotic freedom. Moreover, also the slope of the curves (which on a linear time-scale would be 2δ in the analytical estimate above) *decreases* towards higher energy, i.e. equilibration slows down ! Nevertheless, one also oberves that the slope increases with time, unlike in the analytical estimate where $\delta = const$. Thus, once $\phi \sim 1$, collisions "accumulate" and lead to faster equilibration than a simple extrapolation using the slope at τ_0. For example, at BNL-RHIC energy we obtain $\tau_{th} \approx 0.5$ fm, about a factor 3 less than before.

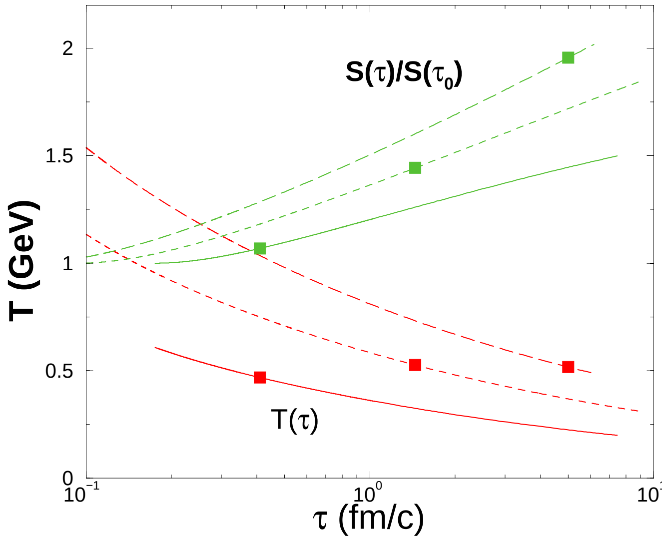

FIG. 3. The temperature and entropy of the minijets corresponding to a state in equilibrium at the same comoving energy density. The squares mark the times where the equilibration condition (0.7) is fulfilled, see also Fig. 2. The curves correspond to center of mass energies $E_{cm} = 100A$ GeV (full lines), $E_{cm} = 2700A$ GeV (short-dashed lines), and $E_{cm} = 25000A$ GeV (long-dashed lines), and $A = 200$ nuclei.

In Fig. 3 we show the time-evolution of the nominal temperature and entropy, calculated as if the partons were in kinetic equilibrium at the corresponding time. The squares mark the times were scattering and expansion rates become equal. As already indicated by the analytical estimate above, the entropy (normalized to unity at the initial time) at τ_{th} *increases* with energy and $p_{\rm sat}$, confirming that the initial state departs from local equilibrium. The initial expansion rate is too large to allow the momentum distribution to stay isotropic (which is the initial condition for this calculation). Nevertheless, the reheating temperature is not decreasing towards higher energy but is more or less constant; a slightly more optimistic prediction than the one above. We see however that the initial "temperature" T_0, which of course grows $T \simeq p_{\rm sat}/2$ [8], is by far not the temperature where (possibly viscous) hydrodynamic evolution sets in.

The evolution of the transverse energy, a basic calorimetric observable, represents another way of showing that the initial stage departs from the hydrodynamic limit as energy increases. E_T at rapidity $\eta = 0$ is given by

$$\frac{E_T(\tau)}{E_T(\tau_0)} = \frac{\tau\epsilon}{\tau_0\epsilon_0} \ . \qquad (0.23)$$

In hydrodynamics the longitudinal expansion performs mechanical work, therefore decreasing the transverse energy [15,18]. In other words, if $\delta > 0$, the quantity (0.23) is obviously decreasing with time, as shown in Fig. 4. In line with the previous observations, the longitudinal expansion performs less and less work, i.e. it approaches free streaming as energy increases. For comparison, we have also performed a calculation where we increased the scattering rate by hand by a factor of four (otherwise same conditions as the other full lines, i.e. BNL-RHIC energy and $A = 200$). In this case E_T follows already rather closely the hydrodynamic predictions, in particular if dissipative corrections are included (relativistic Navier-Stokes theory).

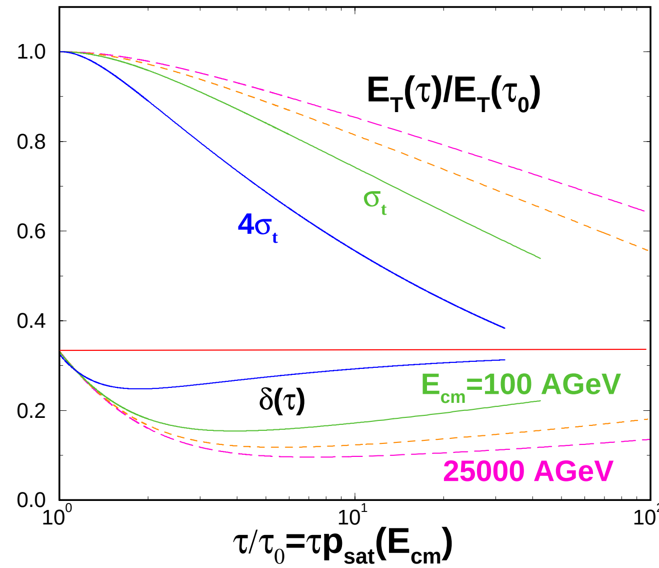

FIG. 4. Time evolution of the transverse energy and the parameter δ which measures the correction to free streaming. Center of mass energies $E_{cm} = 100A$ GeV (full lines), $E_{cm} = 2700A$ GeV (short-dashed lines), $E_{cm} = 25000A$ GeV (long-dashed lines), and $A = 200$ nuclei. The curves labeled $4\sigma_t$ correspond to a calculation with four times larger scattering rate.

The curves at the bottom of Fig. 4 show the time-evolution of $\delta(\tau)$. The horizontal line at $1/3$ is the isentropic velocity of sound of an ultrarelativistic fluid, i.e. the ideal hydrodynamics limit. Initially, $\delta = 1/3$ but this is just an accidental consequence of the particular space-momentum correlation build into the initial condition (i.e. the isotropic initial momentum distribution in the comoving frame). The minijets *do not* evolve hydrodynamically (the less the higher the scale $p_{\rm sat}$) as can be seen from the ratio $\Gamma_{scatt}/\Gamma_{exp}$ in Fig. 2, from the entropy increase in Fig. 3, from the flatness of E_T in Fig. 4, and finally from the fact that initially δ decreases very rapidly with time. Such a behavior can not be described hydrodynamically, not even within Navier-Stokes theory. The fact that δ decreases initially is a consequence of the fact that the expansion rate is too large and the parton rescattering in the limit $p_{\rm sat} \to \infty$ can not compete with it because it contains powers of the coupling $\alpha_s(p_{\rm sat}) \to 0$ (asymptotic freedom). The higher the scale $p_{\rm sat}$ the stronger the initial drop of δ, such that

δ asymptotically approaches zero. At later times, when $\Gamma_{scatt}/\Gamma_{exp} > 1$, the scattering rate catches up with the expansion rate and δ slowly approaches the isentropic velocity of sound of an ultrarelativistic fluid, $c_s^2 = 1/3$. In the calculation with ad-hoc enhancement of the scattering rate the turn-over happens already at $\tau/\tau_0 \approx 2$, and relativistic Navier-Stokes theory is probably a reasonable approximation to the subsequent evolution. On the other hand, without ad-hoc increase of Γ_{scatt}, and for energies far above BNL-RHIC energy ($E_{cm} = 100A$ GeV), δ drops to less than $c_s^2/2$ and (viscous) hydrodynamics seems to apply only at times $\tau > (100-500)\tau_0$.

In summary, we have calculated the reheating temperature (i.e. the temperature at the time of kinetic equilibration) in high-energy hadronic collisions, assuming saturation of the number of produced secondary partons in the central rapidity region. As a consequence of the fact that within this model the hard scale *evolves* with center of mass energy, asymptotic freedom together with the increasing Bjorken/Hubble expansion rate force the minijet plasma into the Knudsen limit (close to free streaming). Within the uncertainties introduced by various approximations and assumptions, it seems that the reheating temperature is essentially constant above BNL-RHIC energy.

ACKNOWLEDGMENTS

A.D. thanks Larry McLerran for helpful criticism and discussions on thermalization aspects, and Kimmo Tuominen for useful remarks regarding saturation. We acknowledge support from the DOE Research Grant, Contract No. De-FG-02-93ER-40764.

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